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THE FINITE RESIDUAL MOTION OF A DAMPED FOUR-DEGREE-OF-FREEDOM VIBRATING SYSTEM

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In Figures 1 and 2 are shown two systems which oscillate in the horizontal direction. The four bodies each have unit mass, the springs have unit stiffness, and the dashpots have constant c. One of the systems oscillates indefinitely, while all oscillations are eventually damped out for the other one. Which one oscillates indefinitely?

The undamped equations of motion are

$$\ddot{x}_1 = -2x_1 + x_2, \qquad \ddot{x}_2 = x_1 - 2x_2 + x_3, \tag{1, 2}$$

$$\ddot{x}_3 = x_2 - 2x_3 + x_4, \qquad \ddot{x}_4 = x_3 - 2x_4. \tag{3, 4}$$

From the symmetry of the system, the undamped mode shapes must be either symmetric, or antisymmetric with respect to the centre of the system.

Equations (1)–(4) reduce to

$$\ddot{x}_1 = -2x_1 + x_2, \qquad \ddot{x}_2 = x_1 - x_2, \tag{5, 6}$$

if $x_3 = x_2$ and $x_1 = x_4$. The corresponding symmetric undamped mode shapes are plotted in Figures 3(a) and (b) (see reference [1]).

If $x_1 = -x_4$ and $x_2 = -x_3$, the corresponding antisymmetric undamped mode shapes are plotted in Figures 3(c) and (d).

Guided by the undamped mode shapes, we make the variable changes

$$\begin{aligned} x_1 &= A + B + D + E, \\ x_2 &= \frac{A(\sqrt{5}+1)}{2} + \frac{B(1-\sqrt{5})}{2} - \frac{D(1+\sqrt{5})}{2} + \frac{E(\sqrt{5}-1)}{2}, \\ x_3 &= \frac{A(\sqrt{5}+1)}{2} + \frac{B(1-\sqrt{5})}{2} + \frac{D(1+\sqrt{5})}{2} + \frac{E(1-\sqrt{5})}{2}, \\ x_4 &= A + B - D - E. \end{aligned} \tag{7}$$

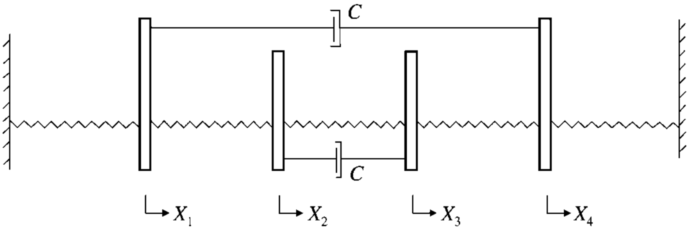

Figure 1. System A.

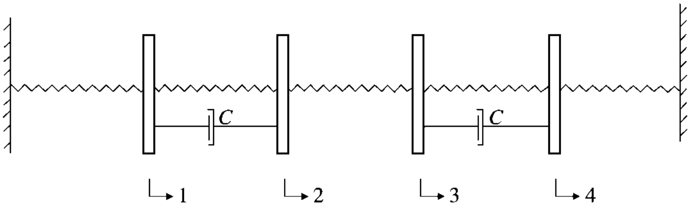

Figure 2. System B.

The kinetic energy is

$$T = \frac{1}{2}\sum_{i=1}^{4} \dot{x}_i^2 = \frac{\dot{A}^2(5+2\sqrt{5})}{2} + \frac{\dot{B}^2(5-2\sqrt{5})}{2} + \frac{\dot{D}^2(5+2\sqrt{5})}{2} + \frac{\dot{E}^2(5-2\sqrt{5})}{2}. \quad (8)$$

The potential energy is

$$V = \tfrac{1}{2}(x_1^2 + (x_2 - x_1)^2 + (x_3 - x_2)^2 + (x_4 - x_3)^2 + x_4^2)$$
$$= A^2(10 - \sqrt{5}) + B^2(10 + \sqrt{5})$$
$$+ D^2(20 + 5\sqrt{5}) + E^2(20 - 5\sqrt{5}). \quad (9)$$

For System A, we then have the dissipation function

$$F = \frac{c}{2}(\dot{x}_3 - \dot{x}_2)^2 + \frac{1}{2}(\dot{x}_4 - \dot{x}_1^2)$$
$$= \frac{c}{2}(\dot{D}^2(10 + 2\sqrt{5}) + \dot{E}^2(10 - 2\sqrt{5})). \quad (10)$$

Lagrange's equations,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} + \frac{\partial F}{\partial \dot{q}_k} = 0, \quad (11)$$

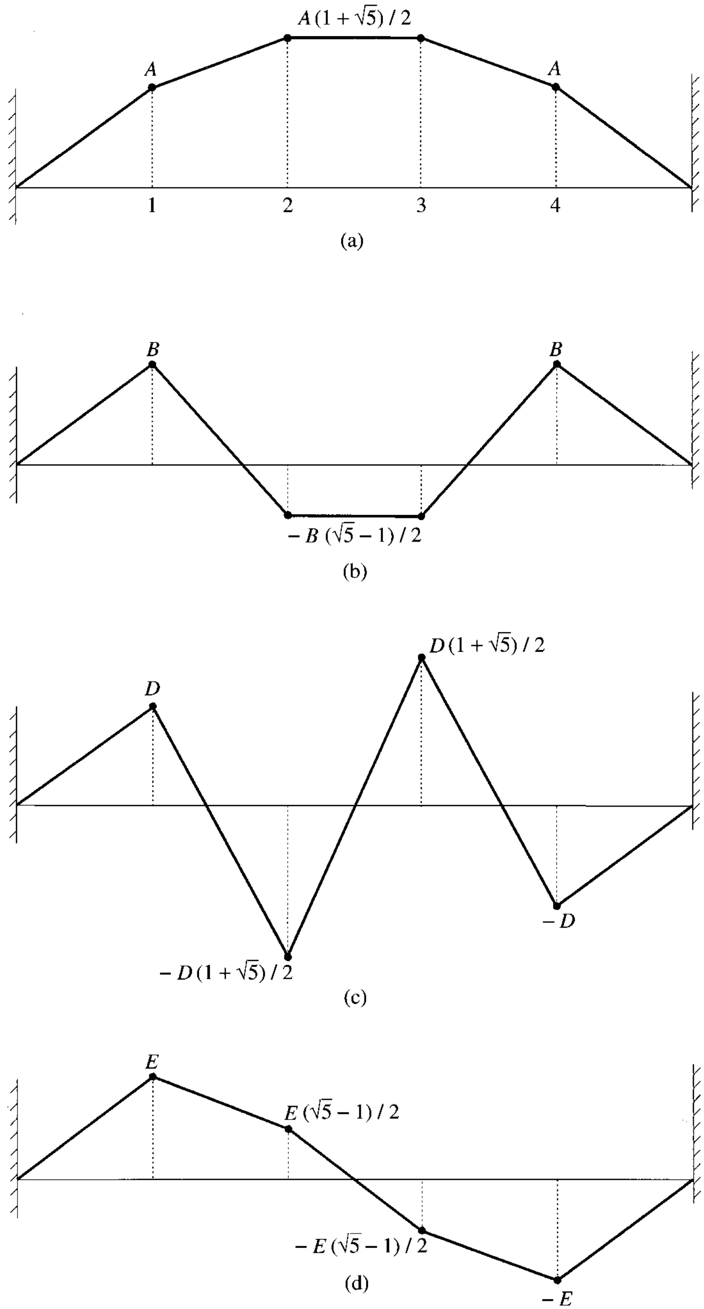

Figure 3. (a) Mode shape A, (b) mode shape B, (c) mode shape D, (d) mode shape E.

where $q_1 = A$, $q_2 = B$, $q_3 = D$, $q_4 = E$ and $L = T - V$, then yield

$$\ddot{A}(5 + 2\sqrt{5}) + A(10 - \sqrt{5}) = 0, \tag{12}$$

$$\ddot{B}(5 - 2\sqrt{5}) + B(10 + \sqrt{5}) = 0, \tag{13}$$

$$\ddot{D}(5+2\sqrt{5})+c\dot{D}(10+2\sqrt{5})+D(20+5\sqrt{5})=0, \tag{14}$$

$$\ddot{E}(5-2\sqrt{5})+c\dot{E}(10-2\sqrt{5})+E(20-5\sqrt{5})=0. \tag{15}$$

Equations (12) and (13) indicate undamped motion and equations (14) and (15) indicate damped motion. Hence for general initial values of x_i and $\dot{x}_i$ ($i = 1, 2, 3, 4$), finite residual motion remains for System A.

The dissipation function $\frac{1}{2}c(\dot{x}_l - \dot{x}_m)^2$ for a dashpot located between l and m indicates that finite motion remains only for $l = 2$, $m = 3$, and $l = 1$, $m = 4$. The other remaining locations result in all motion being damped out. Hence all motion is damped out for System B.

In references [2, 3] a procedure using vibration and control theory was used to determine whether finite residual motion remained or whether all motion was damped out. These systems had two degrees of freedom [2, 4, 5] and three degrees of freedom [1, 3] respectively.

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